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Brief Communication

ON THE STATIC INSTABILITY OF FLEXIBLE PIPES CONVEYING FLUID

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1. INTRODUCTION

The problem of fluid flow through flexible pipes has received a good deal of attention in the research literature. Païdoussis & Issid (1974) introduced the basic governing differential equations, where it was shown that the system could be subjected to both divergence and flutter instabilities. Laura et al. (1987) investigated bending motion of a simply supported pipeline conveying fluid using a power series method to solve the associated governing equations. Mishra & Upadhyay (1987) used a cylindrical shell model to account for the rotary inertia and shear deformation effects. Concerning system optimization, Borglund (1998) formulated the minimal structural-mass design problem for a fixed critical flow speed. Analysis was performed using the finite element method to solve the associated equation of motion of a cantilevered configuration.

Based on the fact that an exact solution for a uniform pipe is available and well established, this study presents a mathematical model for determining the exact critical flow velocity of a pipeline composed of uniform modules. Design parameters include the wall thickness and the length of each module. As a case study, the developed model is applied to a simply supported pipeline consisting of two, three, and more modules. Clear design charts are given showing the functional behavior of the critical flow velocity with the selected design parameters.

2. ANALYSIS AND MATHEMATICAL FORMULATION

For the kth module of the pipeline shown in Fig. 1, the governing differential equation for the case of static instability (Païdoussis & Issid 1974) can be cast in the following non-dimensional form:

$$w'''' + \lambda_k^2 w'' = 0, \quad \lambda_k = U_k\sqrt{\frac{A_k}{I_k}} = \frac{UA}{\sqrt{A_k I_k}}, \quad k = 1,\ 2, \ldots,\ N_m, \tag{1}$$

which is valid over the entire module length, i.e., $0 \le \bar{x} \le L_k$, where $\bar{x} = x - x_k$. It is noted that $U_k A_k = UA$; w is the bending deflection, U the critical flow velocity, A the maximum cross-sectional area, and N_m the total number of modules composing the pipeline. The various parameters are nondimensionalized by the associated values of a reference uniform pipe having the same total length and material properties (see Table 1). Equation (1) has an exact solution of the form

$$w(\bar{x}) = B_1 + B_2\bar{x} + B_3 \sin \lambda_k \bar{x} + B_4 \cos \lambda_k \bar{x}, \tag{2}$$

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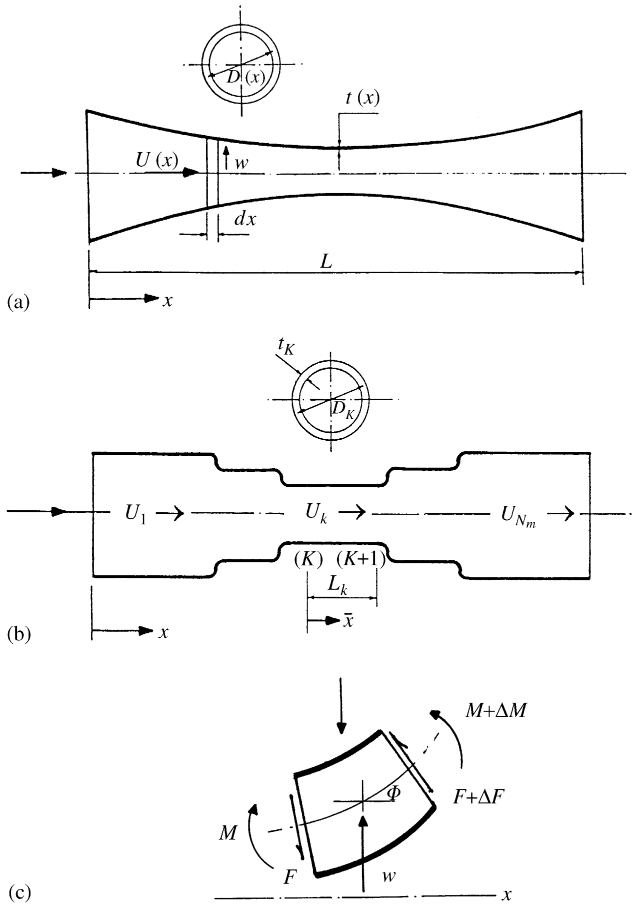

Figure 1. General configuration of a tubular pipe conveying fluid: (a) continuous model; (b) discrete multi-module model; (c) equilibrium of an element dx.

TABLE 1

Nondimensional quantities

Quantity	Notation	Nondimensionalization*
Axial coordinate	X	$x \leftarrow x/L_o$
Module length	L_k	$L_k \leftarrow L_k/L_o$
Wall thickness	t_k	$t_k \leftarrow t_k/t_o$
Mean diameter	D_k	$D_k \leftarrow D_k/D_o$
Cross-sectional area	$A_k(=\pi D_k^2/4)$	$A_k \leftarrow A_k/A_o(=D_k^2)$
Second moment of inertia	$I_k(\approx \pi D_k^3 t_k/8)$	$I_k \leftarrow I_k/I_o(=D_k^3 t_k)$
Transverse displacement	W	$w \leftarrow w/L_o$
Bending moment	M	$M \leftarrow M^*(L_o/EI_o)$
Shearing force	F	$F \leftarrow F^*(L_o^2/EI_o)$
Axial flow velocity	U_k	$U_k \leftarrow U_k^*(\rho A_o L_o^2/EI_o)^{1/2}$
Structural mass	M_s	$M_s \leftarrow M_s/M_o \left(=\sum_{k=1}^{N_m} D_k t_k L_k\right)$

*Reference values: L_o = length, t_o = wall thickness, D_o = mean diameter, $A_o = \pi D_o^2/4$, $I_o \approx \pi D_o^3 t_o/8$, M_o = pipe mass $= \rho_p \pi D_o t_o L_o$, ρ = fluid density, ρ_p = material density, where the B_is are constants to be determined by applying appropriate boundary conditions.

Applying the transfer matrix technique (Nagai & Hayama 1991), the state vector, Z_k, at any joint (k) within the pipeline is defined as follows:

$$\mathbf{Z}_k^{\mathrm{T}} = [\mathrm{w},\ \varphi,\ M,\ F]_k = [\mathrm{w},\ -\mathrm{w}', -\mathrm{Iw}'', -\mathrm{Iw}''']_k. \tag{3}$$

At two successive joints (k) and ($k+1$) the state vectors are related to each other by the matrix equation

$$\mathbf{Z}_{k+1} = [\mathbf{T}_r]_k \mathbf{Z}_k, \tag{4}$$

where $[\mathbf{T}_r]_k$ is a square matrix of order 4×4 known as the transmission or transfer matrix of the kth module. The final derived form is

$$[\mathbf{T}_r]_k = \begin{bmatrix} 1 & -L_k & (C_k - 1)/I_k\lambda_k^2 & \left(\dfrac{S_k}{\lambda_k} - L_k\right)/I_k\lambda_k^2 \\ 0 & 1 & S_k/I_k\lambda_k & (1 - C_k)/I_k\lambda_k^2 \\ 0 & 0 & C_k & S_k/\lambda_k \\ 0 & 0 & -\lambda_k S_k & C_k \end{bmatrix}, \tag{5}$$

where $C_k = \cos\lambda_k L_k$ and $S_k = \sin\lambda_k L_k$. For a pipeline built from N_m uniform modules, equation (4) can be applied at successive joints to obtain

$$\mathbf{Z}_{N_m+1} = [\mathbf{T}]\mathbf{Z}_1, \tag{6}$$

where $[\mathbf{T}]$ is called the overall transmission matrix formed by taking the products of all the intermediate matrices of the individual modules. Therefore, applying the boundary conditions and considering only the nontrivial solution, the resulting characteristic equation can be solved for the critical flow velocity. The main focus of the present study will be on the case of simply supported pipelines. The more general case of an elastically restrained pipeline will be investigated in detail by the authors in a future study.

3. APPLICATIONS AND COMPUTATIONAL RESULTS

For a simply supported uniform pipeline consisting of one module, the nondimensional critical flow velocity is given by $U = \pi\sqrt{D_1 t_1}$. It is obvious that there is no way to increase U above its principal value of π without the penalty of increasing the structural mass ($M_s = D_1 t_1$).

Figure 2 shows the functional behavior of the critical flow velocity of a two-module model having unit nondimensional diameter and structural mass (i.e., the pipe model has the same diameter and mass as that of the reference design). It is seen that the absolute maximum value of the critical velocity is close to $3{\cdot}238$ ($> \pi$), where the optimum design point is $(t_k,\ L_k) = (0{\cdot}39,\ 0{\cdot}135),\ (1{\cdot}095,\ 0{\cdot}865)$.

Several other cases of study have been implemented and investigated. Results indicated that, for a simply supported pipeline, good patterns must be symmetrical about the mid-span point. Therefore, it can be easier to cope with symmetrical configurations, which reduce computational effort significantly, and the total number of design variables to half. In this case, the boundary conditions become $w(0) = w''(0) = 0$ and $w'(1/2) = w'''(1/2) = 0$, and the associated characteristic equation for calculating U takes the form [refer to equation (6)]

$$T_{22}T_{44} - T_{24}T_{42} = 0. \tag{7}$$

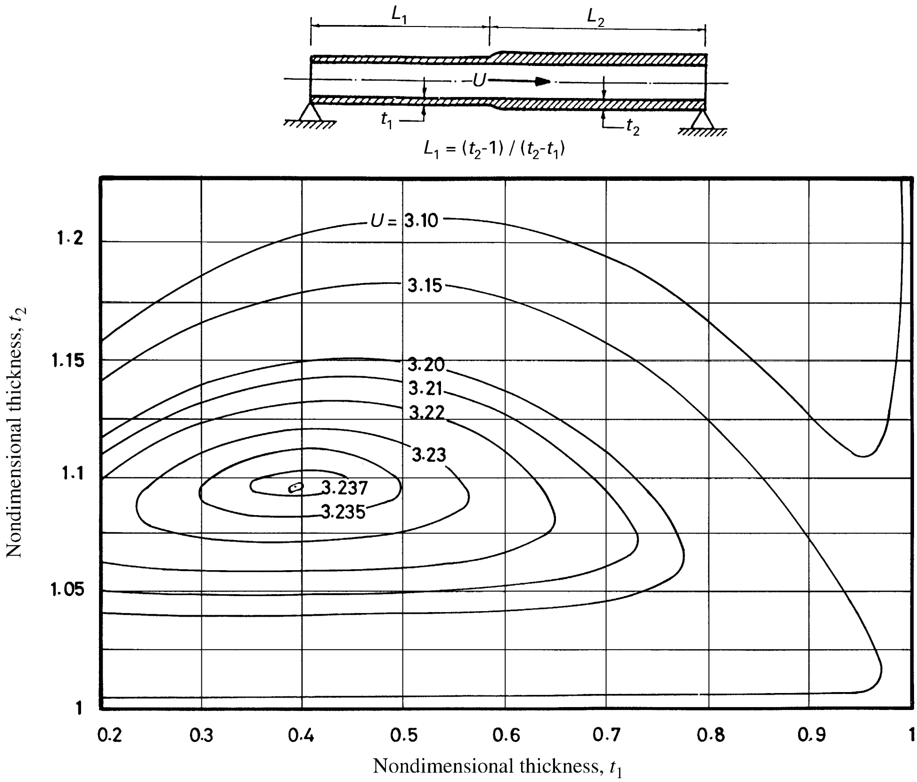

Figure 2. Behavior of the critical flow velocity for two-module model.

Figure 3 depicts the optimum zone of a symmetrical three-module model having a unit nondimensional diameter and structural mass.

The final results for symmetrical patterns are summarized in Table 2, where the subscript s refers to symmetry about the mid-span point. It is important to mention here that the resulting optimal solutions depend significantly on the preassigned lower limits imposed on the wall thickness of the pipeline. Such limits are usually related to considerations of local instability that might be caused by buckling.

4. CONCLUSIONS

The functional behavior of the critical flow speed through a simply supported multi-module pipeline is investigated in detail. The effective design variables are chosen to be the wall thickness and length of each module. Extensive computer implementations have proved that the critical speed, even though an implicit function of the design variables, is well behaved, monotonic and defined everywhere in the selected design space, which ensure the exact determination of the static stability boundary. A useful conclusion is the possibility of selection of the module length as a main design variable. Investigators who employ the finite element method ignore this variable. Good symmetrical configurations have shown that the wall thickness ought to decrease near the boundaries, while it takes the maximum allowable values at the middle portion of the pipeline. Results have also

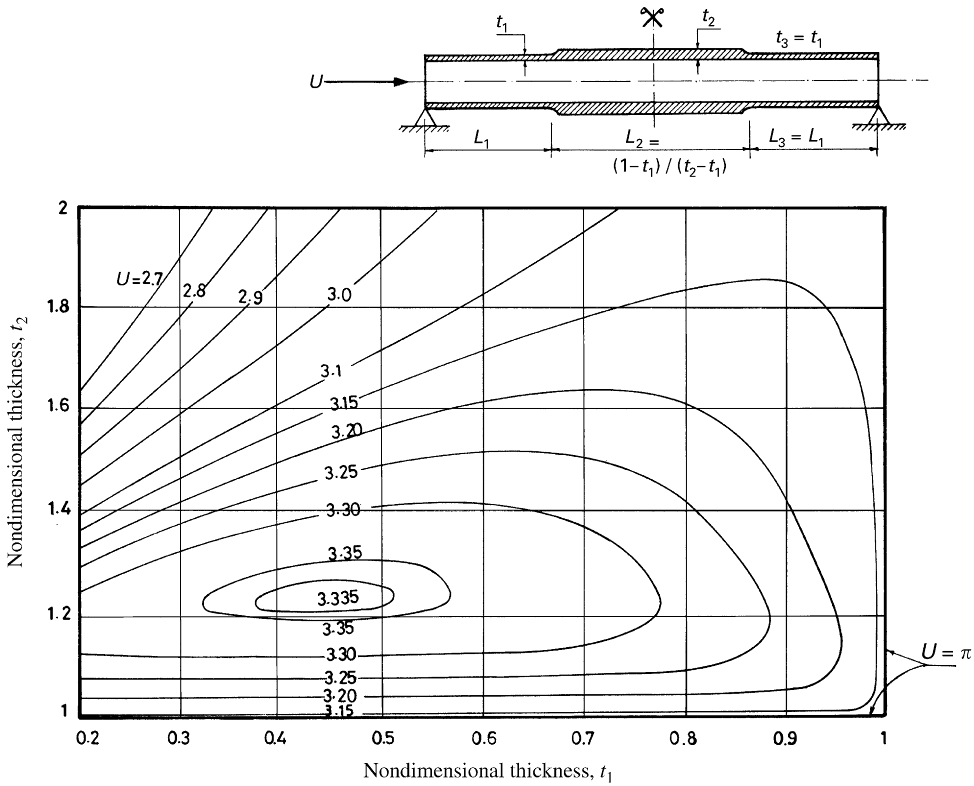

Figure 3. Optimum zone for symmetrical three-module model.

TABLE 2

Optimal patterns of simply supported pipelines

N_m	$[(t_k, L_k)]$	U_{max}	Gain (%)
3	$[(0{\cdot}45, 0{\cdot}15625), (1{\cdot}25, 0{\cdot}34375)]_s$	3·3590	6·9
5	$[(0{\cdot}25, 0{\cdot}075), (0{\cdot}75, 0{\cdot}15), (1{\cdot}3409, 0{\cdot}275)]_s$	3·4121	8·6
7	$[(0{\cdot}15, 0{\cdot}05), (0{\cdot}5, 0{\cdot}075), (0{\cdot}9, 0{\cdot}125), (1{\cdot}37, 0{\cdot}25)]_s$	3·4332	9·3

indicated that the increase in the number of modules results in higher values of the critical flow speed, and consequently, the overall stiffness level. However, care should be taken for the increased cost of connections. Finally, the present exact analysis saves much of the computer time required by the finite element and other discretized approximate methods.

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